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2812 [1920, 81]. Proposed by C. N. SCHMALL, New York City.

If $F(x, y, z)$ be a homogeneous function of x, y, z which becomes $\phi(u, v, w)$ by elimination of x, y, z by means of the equations, $\partial F/\partial x = u$, $\partial F/\partial y = v$, $\partial F/\partial z = w$; show that

$$\frac{\partial F}{\partial u}\bigg/x = \frac{\partial F}{\partial v}\bigg/y = \frac{\partial F}{\partial w}\bigg/z.$$

SOLUTION BY THE PROPOSER.

The result is incorrectly stated. It should read,

$$\frac{\partial \phi}{\partial u}\bigg/x = \frac{\partial \phi}{\partial v}\bigg/y = \frac{\partial \phi}{\partial w}\bigg/z.$$

Let k be the degree of the homogeneous function F. Then by Euler's theorem on homogeneous functions,

$$xu + yv + zw = kF. \tag{1}$$

By partial differentiation of (1) with respect to x, we obtain

$$u + x\frac{\partial u}{\partial x} + y\frac{\partial v}{dx} + z\frac{\partial w}{\partial x} = ku,$$

or

$$x\frac{\partial u}{\partial x} + y\frac{\partial v}{\partial x} + z\frac{\partial w}{\partial x} = (k - 1)u. \tag{2}$$

Similarly, we get

$$x\frac{\partial u}{\partial y} + y\frac{\partial v}{\partial y} + z\frac{\partial w}{\partial y} = (k - 1)v \tag{3}$$

and

$$x\frac{\partial u}{\partial z} + y\frac{\partial v}{\partial z} + z\frac{\partial w}{\partial z} = (k - 1)w. \tag{4}$$

Again,

$$u = \frac{\partial F}{\partial x} = \frac{\partial \phi}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial v}\cdot\frac{\partial v}{\partial x} + \frac{\partial \phi}{\partial w}\cdot\frac{\partial w}{\partial x}, \tag{5}$$

$$v = \frac{\partial F}{\partial y} = \frac{\partial \phi}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial \phi}{\partial v}\cdot\frac{\partial v}{\partial y} + \frac{\partial \phi}{\partial w}\cdot\frac{\partial w}{\partial y}, \tag{6}$$

$$w = \frac{\partial F}{\partial z} = \frac{\partial \phi}{\partial u}\cdot\frac{\partial u}{\partial z} + \frac{\partial \phi}{\partial v}\cdot\frac{\partial v}{\partial z} + \frac{\partial \phi}{\partial w}\cdot\frac{\partial w}{\partial z}. \tag{7}$$

The solution of the system (2), (3), (4) for x, y, z, and of the system (5), (6), (7) for $\partial\phi/\partial u$, $\partial\phi/\partial v$, $\partial\phi/\partial w$ gives

$$\frac{\partial \phi}{\partial u}\bigg/x = 1/(k - 1), \qquad \frac{\partial \phi}{\partial v}\bigg/y = 1/(k - 1), \qquad \frac{\partial \phi}{\partial w}\bigg/z = 1/(k - 1).$$

Whence,

$$\frac{\partial \phi}{\partial u}\bigg/x = \frac{\partial \phi}{\partial v}\bigg/y = \frac{\partial \phi}{\partial w}\bigg/z.$$

2823 [1920, 185]. Proposed by S. A. COREY, Des Moines, Iowa.

Let TQ and PR be diameters of a circle with center O. Bisect TO at X and draw PQ. On PQ erect the perpendicular XW and on PR, the perpendicular QV. Prove that $OX\cdot PV = PW\cdot PQ$

SOLUTION BY EMMA M. GIBSON, Springfield, Mo.

If S is the intersection of XW and PO, the triangle XOS is isosceles (the reader is requested to draw the figure); for,

$$\angle SXO = 90° - \angle XQW = 90° - \angle SPW = \angle PSW = \angle XSO.$$

Hence $OX = OS = SP$. Since the triangles PWS and PVQ are similar,

$$PW/PV = PS/PQ = OX/PQ,$$

and hence

$$OX\cdot PV = PW\cdot PQ.$$